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TRANSITION PROBABILITY IN RESONANCE CONDITION FOR Z=110

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ABSTRACT
The transition probability for different states are found out for the isotopes of z=110.The half-life are also calculated and it is seen that the stable form of isotope is A=281 ,z=110.

KEYWORDS: Resonance condition, selection rules, types of nuclear reactions, decay constant, isotopes of z=110.

INTRODUCTION
The probability of the occurrence of a transition between two quantum states of an atom, nucleus, electron etc. is known as the transition probability. The transition probability corresponding to a transition from an initial unperturbed state to another unperturbed state depends upon the strength of the coupling between the initial and finale state of a system and upon the number of ways the transition can happen i.e the density of the finale states[1].

The transition probability is also called the decay probability and is related to the mean life time of the state. If λ is the transition probability and τ be the mean life time of the state, then the relation is
$\lambda = 1/\tau$

There is the general form of Fermi Golden rule that can apply to atomic transitions, nuclear decay scattering etc. The form is given as $\lambda = \frac{2\pi}{\hbar} |H_{if}|^2 \dot{\rho}_f(E)$, where H_{if} is the matrix element of the transition from initial to the finale state.

A transition will proceed more rapidly if the coupling between the initial and the final states is stronger. The transition probability is also proportional to the density of the final states $\dot{\rho}_f(E)$.
In the case of nuclear reaction, the reaction takes place in two distinct and independent stages.

1. Formation of a compound nucleus C , which survives relatively long time and
2. The disintegration of the compound nucleus into the products of the reaction.

The compound nucleus which is a many body system of strongly interacting particles is formed by the mixture of an incident particle x with a target nucleus X i.e $X + x \rightarrow C^*$ The mode of disintegration of compound nucleus , $C^* \rightarrow Y + y$ is independent of the mode of formation .

Transition probability in resonance condition is nothing but the probability of going the system from one state to the different excited states of the compound nucleus . There is no difference between the compound nucleus and the nuclear resonance[2].

METHOD TO FIND TRANSITION PROBABILITY
Time-dependent perturbation theory is most useful for treating the transitions of quantum systems from one energy level to another. The transition probability corresponding to a transition from an initial unperturbed state $|\Psi_i>$ to another unperturbed state $|\Psi_f>$ is obtained by

$$P_{if}(t) = |<\Psi_f |\dot{U}_I(t,t_i)|\Psi_i>|^2$$

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$= |\langle\Psi_f|\Psi_i\rangle - \frac{i}{\hbar} e^{i\omega fit'} \langle\Psi_f| V(t'))|\Psi_i > dt' + (-\frac{i}{\hbar})2\sum\int_0^t e^{i\omega fit_1}$
$\langle\Psi_f|V(t_1)|\Psi_n\rangle dt_1 \int_0^t e^{i\omega ni\, t_2}\langle\Psi_n|V\hat{}(t_2)|\Psi_i\rangle dt_2 + \ldots..|^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$

Where, $\langle\Psi_f|V\hat{}(t')|\Psi_i\rangle = \langle\Psi_f|e^{iHo}e^{t'}/\hbar\, V\hat{}(t')\, e - {}^{iHo}\, e^{t'}/\hbar|\Psi_i\rangle = \langle\Psi_f|V\hat{}(t')|\Psi_i\rangle e^{i\omega fit'}$

Where, ω_{fi} is the transition frequency between the initial and final levels i and f.

$$\omega_{fi} = E_f - E_i / \hbar$$
$$= \frac{1}{\hbar}(\langle\Psi_f|H_0|\Psi_f\rangle - \langle\Psi_i|H_0|\Psi_i\rangle)$$

V(t) is the external time-dependent perturbation that is small compared to H_0 .The first order probability for $|\Psi_i\rangle \rightarrow |\Psi_f\rangle$ with $i \neq f$ is obtained by terminating equation (1) at the first order in $V_I(t)$.

$$\text{So, } P_{if}(t) = |-\frac{i}{\hbar}\int e^{i\omega fit'}\langle\Psi_f|\ V\hat{}(t')|\Psi_i > dt'|^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

For most of the nuclear physics, the first order is usually sufficient.
We all have the transition probability defined by

$$P_{if}(t) = \frac{2\pi t}{\hbar}|\langle\Psi_f|V\hat{}|\Psi_i\rangle|^2\delta(E_f - E_i)$$

The transition rate which is defined as the transition probability per unit time is given by

$$\mathbf{\Gamma}_{if} = \frac{P_{if}(t)}{t} = \frac{2\pi}{\hbar}|\langle\Psi_f|V\hat{}|\Psi_i\rangle|^2\delta(E_f - E_i) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

To calculate the total transition rate associated with a transition from an initial state $|\Psi_i\rangle$ to a continuum of final states $|\Psi_f\rangle$, let $\dot{\rho}_f(E)$ is the density of final states i.e the number of states per unit energy interval.

The number of final states within the energy interval E_f and E_f+dE_f is equal to $\dot{\rho}_f(E)\, dE_f$ .The total transition rate $\mathbf{w_{if}}$ can be obtained from equation(3).

$$\mathbf{W_{if}} = \int \frac{P_{if}(t)}{t}\, \dot{\rho}_f(E)\, dE_f$$
$$= \frac{2\pi}{\hbar}|\langle\Psi_f|V\hat{}|\Psi_i\rangle|^2 \int \dot{\rho}_f(E)\, \delta(E_f - E_i)dE_f$$

$$\Rightarrow \quad \mathbf{W_{if}} = \frac{2\pi}{\hbar}|\langle\Psi_f|V\hat{}|\Psi_i\rangle|^2\, \dot{\rho}(E_i) \quad \ldots\ldots\ldots\ldots\ldots..(4)$$

This relation is called the Fermi Golden rule.

RESONANCE CONDITIONS

The compound nucleus is the intermediate state formed in a compound nuclear reaction. It is normally one of the excited states of the nucleus formed by the combination of the incident particle and target nucleus.

For the compound nucleus, peaks in the cross section are typical. Each peak is manifesting a particular compound state of nucleus. These peaks and the associated compound nuclei are usually called resonances.

Resonances or particular compound states are mostly created in neutron nuclear reactions but it is by no means restricted to neutron nuclear reactions.

Each nuclear reaction is a transition between different quantum discrete states or energy levels. The discrete nature of energy transitions plays a key role. If the energy of the projectile and the energy of target nucleus is equal to a compound nucleus at one of the excitation states, a resonance can be created and peak occurs in the cross section.

It is obvious that the compound states or resonances are observed at low excitation energies. This is due to the fact, the energy gap between the states is large. At high excitation energy, the gap between two compound states is very small and the widths of resonances may reach the order of the distances between resonances. Therefore, at high energies no resonances can be observed and the cross section in this region is continuous and smooth.

If a target nucleus 'X' is bombarded with particles x, it is sometimes observed that the nuclear reaction takes place with appreciable

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probability only if the energy of the particle 'x' is in the neighbourhood of certain definite energy values. These energy values are reffered as resonance energies. The compound nuclei of these certain energies are reffered to as nuclear resonances.

The narrowest resonances are usually the compound states of heavy nuclei such as fissionable nuclei and thermal neutrons usually in (n , Y) capture reactions.

The transition considered takes place from a definite initial state (X + x) to the different energies of the emitted particle y and different energy states in which the residual nucleus Y is left. It is generally assumed that the particle y is produced in its ground state[3].

EXAMPLES OF COMPOUND NUCLEAR REACTIONS:

In the nuclear transmutation reactions , involving particles of not too high energy, having energy less than 50 M e V, the first stage is the formation of compound nucleus[4].

TRANSMUTATION BY ALPHA PARTICLES:

(a) (α – p) reaction

Rutherford first observed the transmutation of the Nitrogen nucleus by α- particles in which α- particle was captured by a nucleus forming a compound nucleus which disintegrated immediately into a new nucleus by the ejection of a proton. Few of other (α – p) reactions are,

$$^{10}_{5}B + ^{4}_{2}He \longrightarrow ^{13}_{6}C + ^{1}_{1}H + 4.04 \text{ MeV}$$

$$^{19}_{9}F + ^{4}_{2}He \longrightarrow (^{22}_{11}Na)^{*} \longrightarrow ^{22}_{10}Ne + ^{1}_{1}H + 1.58 \text{ MeV}$$

Few (α - p) reactions lead to the formation of radiative isotopes that decay by β- emission. For example

$$^{11}_{5}B + ^{4}_{2}He -> (^{15}_{7}N) * -> ^{14}_{6}C + ^{1}_{1}H + 0.75 \text{ MeV}$$

(α - n) reaction :

The bombardment of Beryllium by α – particles with the subsequent emission of neutrons is one of the (α - n) reactions. Few of other of (α - n) type of nuclear reactions are :

$$^{7}_{3}Li + ^{4}_{2}He -> (^{11}_{5}B) * -> ^{10}_{5}B + ^{1}_{0}n - 2.79 \text{ MeV}$$

$$^{9}_{4}Be + ^{4}_{2}He -> (^{13}_{6}C) * -> ^{12}_{6}C + ^{1}_{0}n + 5.70 \text{ MeV}$$

In many of these reactions the product nuclei are left in excited states. Most of the product nuclei formed by (α - n) reaction are unstable isotopes which then disintegrate with the emission of a positron.

A few of these are given below with their half lives :

$$^{13}_{7}N -> ^{13}_{6}C + \beta^{+}, \text{ T = 9.96 min}$$

$$^{22}_{11}Na -> ^{22}_{10}Ne + \beta^{+}, \text{ T = 64.5 sec}$$

Radiative capture:

It is also possible, for the compound nucleus formed by the capture of an α – particle to go to a more stable configuration without emitting a particle but with emitting a Y – ray photon .

$$^{7}_{3}Li + ^{4}_{2}He -> (^{11}_{5}B) * -> ^{11}_{5}B + h\nu$$

Transmutation by protons:

(p – α) reaction :

$$^{6}_{3}Li + ^{1}_{1}H -> (^{7}_{4}B) * -> ^{4}_{2}He + ^{3}_{2}He + 4.02 \text{ MeV}$$

$$^{9}_{4}Be + ^{1}_{1}H -> (^{10}_{5}B) * -> ^{6}_{3}Li + ^{4}_{2}He + 2.12 \text{ MeV}$$

(p – n) reaction :

The probability of the (p,n) reaction is relatively high if the proton has sufficient energy to penetrate the Coulomb energy.

Few examples are:

$$^{11}_{5}B + ^{1}_{1}H -> (^{12}_{6}C)^{*} -> ^{11}_{6}C + ^{1}_{0}n - 2.76 \text{ MeV}$$

$$^{18}_{8}O + ^{1}_{1}H -> (^{19}_{9}F) * -> ^{18}_{9}F + ^{1}_{0}n - 2.44 \text{ MeV}$$

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Proton capture:

In some cases the compound nucleus is in excited state which returns to its normal state with the gamma ray photon.

Examples of these type of reactions are:-

$$^{7}_{3}Li + ^{1}_{1}H -> (^{8}_{4}Be)* -> ^{8}_{4}Be + Y + 17.25 \text{ MeV}$$

$$^{12}_{6}C + ^{1}_{1}H -> (^{13}_{7}N)* -> ^{13}_{7}N + Y + 1.94 \text{ MeV}$$

TRANSMUTATION BY NEUTRONS

Neutrons have no electric charge and can penetrate positively charged nuclei without any experience of repulsive electrostatic forces , hence neutrons have also been found to be extremely effective.

(n – α) reaction :

In this type of reactions , the capture of slow neutron results in the emission of an alpha particle.

Few examples are :-

$$^{10}_{5}B + ^{1}_{0}n -> (^{11}_{5}B)* -> ^{7}_{3}Li + ^{4}_{2}He + 2.79 \text{ MeV}$$

$$^{6}_{3}Li + ^{1}_{0}n -> (^{7}_{3}Li)* -> ^{3}_{1}H + ^{4}_{2}He + 4.78 \text{ MeV}$$

(n – p) reaction :

In this reaction , the proton in the nucleus is replaced by a neutron . The mass number is not changed but the charge is decreased by one unit .

Few examples are of (n-p) reaction produced by slow neutrons are:-

$$^{3}_{2}He + ^{1}_{0}n -> (^{4}_{2}He)* -> ^{3}_{1}H + ^{1}_{1}H + 0.76 \text{ MeV}$$

$$^{14}_{7}N + ^{1}_{0}n -> (^{15}_{5}N)* -> ^{14}_{6}C + ^{1}_{1}H + 0.63 \text{ MeV}$$

(n – Y) reaction :

The radioactive capture reaction with slow neutrons is probably the most common nuclear process. The product is an isotope of the target nuclide with mass number larger by unity .

$$^{1}_{1}H + ^{1}_{0}n -> (^{2}_{1}H)* -> ^{2}_{1}H + Y + 2.22 \text{ MeV}$$

$$^{2}_{1}H + ^{1}_{0}n -> (^{3}_{1}H)* -> ^{3}_{1}H + Y + 6.26 \text{ MeV}$$

The capture of a neutron increases the neutron to proton ratio. If this ratio in the target nucleus is already near the upper limit of stability , the product nucleus is radioactive , usually emitting β^- particle.

RESONANCE STATES

Most resonant or unstable states result from the strong interaction of a high energy meson (π or K) with a proton . sometimes, resonant states also result from the interaction with a neutron or a deuteron . These states can be assigned mass , charge and spin consistent with the conservation laws . The evidence for the formation of resonant states , also referred to as resonance particles or resonances . A resonance interaction occurred between the proton and the pion , much as the resonance capture of a neutron by a nucleus was associated with a maximum in the cross section . It is for this reason that the term resonance or resonant state came into general use . At the present time , the name is applied to a particle state of very short life (~ 10^{-23}sec) which decays into two or more particles . Because of their extremely short lifetimes resonant states are described as unstable particles .

In spite of their extremely short lives , two general methods have been used for detecting resonances . One is to measure the scattering cross section as a function of energy of the incident meson .

The average lifetime of the resonance , equal to the uncertainty in the time, $\tau = \Delta t = \frac{\hbar}{\Delta E}$, where ΔE is the uncertainity in energy which is equal to the full width of the peak at half the maximum height .

Suppose the peak width at half maximum is found to be 100 MeV , the mean life of the resonant state is $\frac{6.58}{100} * 10^{-22} = 6.6 * 10^{-24}$sec.

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The scattering cross sections of positive and negative pions by protons are shown in the above figure . The upper curve shows that the maximum of the resonant peak is at a pion energy of 195 Mev in the lab system . The full width of half maximum is 120 MeV , corresponding to the life time of $5.5*10^{-24}$ sec . This short life indicates decay by the strong interaction .

Present works:

Let's consider the case of transition of a super heavy nucleus of Darmstadtium (Ds) having atomic number 110 and atomic mass number 279 to a daughter nucleus of Roentgenium (Rg) having atomic number 111 and atomic mass number 279 under beta decay. Both of them are isotopes having same mass number. The beta decay reaction can be expressed as follows

$$^{279}_{110}Ds \;\; -> \;\; ^{279}_{111}Rg \; + \; ^{0}_{-1}e \; + \; \bar{\nu}$$

Where $\bar{\nu}$ is the anti-neutrino .

Using the shell-model configuration and considering a transition of $^{279}_{110}Ds$ from its first excited state to the ground state of $^{279}_{111}Rg$, the initial and final states are $4P_{3/2}$ to $3P_{1/2}$. The transition is Gamow-Teller type and the selection rules are $\boldsymbol{\Delta J = 0,+1,-1}$ and $\boldsymbol{\Delta}\pi = 0$,where J is the total angular momentum and π is the parity . In this case parity remains conserved.

The transition probability or the decay rate is given as below

$$\lambda = \frac{2\pi}{\hbar}|< \Psi_f|\hat{V}|\Psi_i>|^2 \, \dot{\rho}\,(E_i)$$

Where , $|< \Psi_f|\hat{V}|\Psi_i>|^2 = |H_{fi}|^2$

So, $\lambda = \frac{2\pi}{\hbar}|H_{fi}|^2 \, \dot{\rho}\,(E_i)$

The above expression can also be written as follows

$$\lambda = \frac{1}{30} B E_0^5$$

Where $B = g^2|M_{fi}|^2/2\pi^3 c^6 \hbar^7$

$$E_0 = \Delta E - 2m_e c^2$$

M_{fi} is the transition matrix element, $\dot{\rho}\,(E_i)$ is density of states.

'g' has the experimental value of $8.95 * 10^{-44}$ MeV . cm^3 and

$B = 3.36 * 10^{-3}$ $MeV^{-5}s^{-1}$

Here M_{fi} is Gamow-Teller transition matrix and for this type of transition from $3/2^-$ to $½^-$, the transition is from (l+1/2) to (l-1/2) with l = 1 .

The G-T transition matrix then becomes

$$|M_{GT}|^2 = \frac{4l}{2l+1} = \frac{4}{3}$$

Then the half-life is given as

$$T_{1/2} = \ln 2/\lambda$$

The half-life time versus atomic mass of different isotopes of Z=110

Taking different isotopes of Darmstadtium and to calculate the transition probability and half life of decay of those isotopes,

For Ds-281 isotope, the transition is from $3/2^-$ to $½^-$ and the transition matrix $M_{fi} = 4/3$ with l = 1.

For Ds-279 isotope, the transition is from $3/2^-$ to $½^-$ and $M_{fi} = 4/3$ with l = 1.

For Ds-277 isotope, the transition is from $½^+$ to $½^-$ and $M_{fi} = 5/3$ with

$M_{fi} = (J_f+1)/J_f$, where $l_2 = 1$.

For Ds-279 isotope , the transition is from $3/2^+$ to ½- and $M_{fi} = 4/3$ with l =1.

For Ds-281 isotope , the transition is from $3/2^+$ to $½^-$ and $M_{fi} = 4/3$ with l =1.

Tabulation

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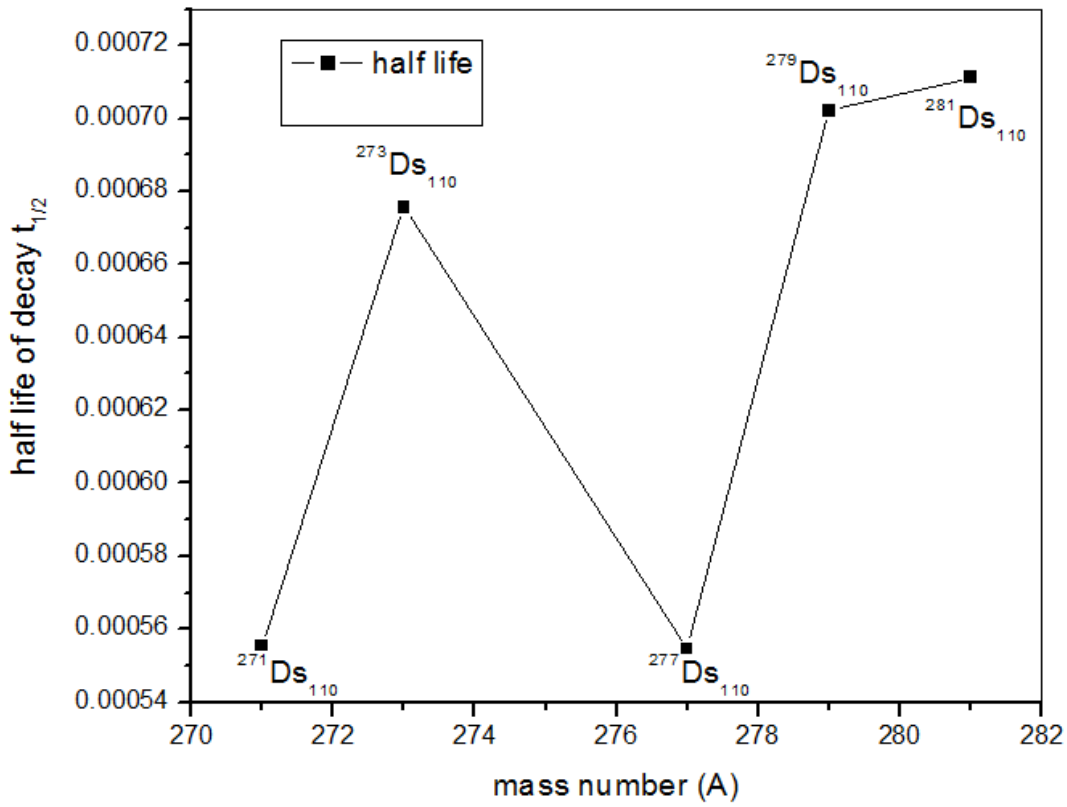
ISOTOPES	TRANSITION PROBABILITY	HALF-LIVES
Ds-271	1247.83 S	$5.554*10^{-4}$ sec
Ds-273	1025.95 S	$6.756*10^{-4}$ sec
Ds-277	1249.87 S	$5.545*10^{-4}$ sec
Ds-279	987.21 S	$7.021*10^{-4}$ sec
Ds-281	974.56 S	$7.112*10^{-4}$ sec

Figure Caption:
The half life period versus atomic mass of different isotopes of z=110



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RESULT AND DISCUSSION

From the above figure drawn between the half-life of decay of the isotopes of z=110 and mass number ,it is seen that the half-life is maximum for the atomic mass A=281 .The stable form of Ds occurs for A=281 whereas unstability appears for A=271 & 277.The different isotopes are having applications in astrophysics ,which are in research level.

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